Core Mathematics C4 For Edexcel Advanced Level

Paper J
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

Published by Elmwood Press
80 Attimore Road
Welwyn Garden City
Herts. AL8 6LP

Tel. 01707 333232

1. Oil is poured onto a flat surface and forms a circular film. The area of the circle increases at a steady rate of 40 $\text{cm}^2\ \text{s}^{-1}$.

(a) Find, in terms of π, the radius of the circle 10 seconds after the pouring of oil commences.

(2)

(b) Find the rate of increase of the radius of the film at this instant. Give your answer in $\text{cm}\ \text{s}^{-1}$ correct to two significant figures.

(4)

---

2. A curve is given by the parametric equations

$$x = 2\sqrt{t}, \qquad y = t^2 - 2.$$

The region below the curve and above the x-axis between the points where $t = 4$ and $t = 9$ has area A.

(a) Show that

$$A = \int_4^9 \left(t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}\right) \mathrm{d}t.$$

(4)

(b) Hence find the value of A.

(3)

---

3. (a) Express

$$\frac{8x + 1}{(2x - 1)(x + 2)}$$

(3)

in partial fractions.

(b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(8x + 1)y}{(2x - 1)(x + 2)},$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$.

(7)

---

4. $$f(x) = \sqrt{\frac{1+x}{1-x}}$$

(*a*) By writing f(*x*) in the form $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$, expand f(*x*) in ascending powers of x up to and including the x^2 term. **(4)**

(*b*) State the values of x for which the expansion is valid. **(1)**

(*c*) By taking $x = \frac{1}{10}$, show that $\sqrt{11}$ is approximately $\frac{663}{200}$ **(3)**

---

5. (*a*) (*i*) Use the identity for $\cos(A+B)$ to prove that $\cos 2A = 2\cos^2 A - 1$ **(2)**

(*ii*) Hence find $\int \cos^2 x \, dx$. **(3)**

(*b*) By using the substitution $x = 2\sin\theta$, show that

$$\int_0^{\sqrt{3}} \sqrt{4-x^2}\, dx = \frac{\sqrt{3}}{2} + \frac{2\pi}{3}.$$ **(5)**

---

6. Points A and B have position vectors $\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix}$ respectively, relative to an origin O.

The line l passes through A and the line m passes through B.

The lines l and m intersect at the point C whose position vector is $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(*a*) Find equations for the lines l and m. **(5)**

(*b*) Find the position vector of the point where l meets the xz plane. **(2)**

(*c*) Calculate the size of $\angle OAB$, giving your answer to the nearest degree. **(3)**

7.

Figure 1

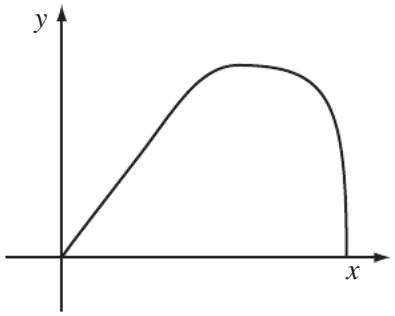

Figure 1 shows the graph of $y = x\sqrt{10 - x^2}$, $0 \leq x \leq \sqrt{10}$.

(*a*) Show that the coordinates of the turning point of the curve are $(\sqrt{5}, 5)$. **(5)**

(*b*) Use the substitution $u = 10 - x^2$ to find the exact area enclosed between the curve and the x-axis. **(7)**

---

8. In a sample of radioactive material, the rate of decay of the number N of radioactive nuclei is proportional to the number of radioactive nuclei present after t days.

(*a*) Write down a differential equation relating N and t. **(2)**

(*b*) Show that the general solution may be written as $N = Ae^{-kt}$, where A and k are positive constants. **(5)**

Given that initially the value of N is 5×10^{12} and that when $t = 10$, $N = 2 \times 10^{11}$,

(*c*) find the values of A and k, **(3)**

(*d*) find the number of radioactive nuclei present when $t = 40$. **(2)**

END **TOTAL 75 MARKS**